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Production of baryon asymmetry of the universe at the electroweak era

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Abstract: We review the elements which enter the calculation of baryon asymmetry at the electroweak scale. We assume the bubble wall created during the phase transition to be sufficiently thin and show that like the (heavy) t quark, the (light) b quark can also produce the observed baryon asymmetry, provided the CP violation within the wall is about two orders of magnitude larger for b quark compared to that for the t quark.

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1 Introduction

It pleases me very much to give this talk in this Meeting honouring Professor Haridas Banerjee. For, although he never worked on this topic, he encouraged me much to work on problems of the early universe. May I also take this opportunity to recall that I owe him a great deal not only in Physics but also in personal life.

The subject of electroweak baryogenesis goes back to 1976 when t'Hooft [1] showed that the standard electroweak theory violates baryon (lepton) number conservation due to transitions between sectors built on gauge invariant vacuua, labelled by a topological index. Being a tunneling process mediated by the instanton, such transition rates are exceedingly small, however. But in 1985 Kuzmin, Rubakov and Shaposhnikov [2] argued that such transitions could be unsuppressed in the early universe, when thermal transitions could take place over the barrier between these sectors.

Furthermore, if the electroweak phase transition is of first order, the motion of the wall of the bubbles nucleated in the unbroken medium would produce departure from thermal equilibrium. C and CP violation is also present in the electroweak theory, originating from the interaction of quarks with the Higgs field. So all the Shakharov conditions [3] for production of baryon asymmetry could be met in the electroweak era. Thus the stage was set for model building to reproduce the observed baryon asymmetry of the universe.

The initial estimate of Shoposhinikov [4] of too low a baryon asymmetry in the standard model led Nelson et al [5] to consider nonminimal extensions of the standard model, where a much bigger source of CP violation is generally available than that provided by the CKM matrix. In their model a CP-odd charge is separated through reflection and transmission of fermions by the bubble wall. It is then converted into an asymmetry in the baryon number by the sphaleron process outside the bubble.

Meanwhile Shaposhnikov himself [6] found that quark mixing effects at high temperature could avoid his earlier discouraging estimate for baryon asymmetry in the minimal standard model. The temperature dependent large effective mass was taken into account. Further he considered a direct separation of baryon number by the bubble wall rather than of some other CP-odd charge. The details were worked out by Farrar and Shaposhnikov [7], reproducing the observed magnitude of baryon asymmetry to within theoretical uncertainties related to quark propagation in the electroweak plasma.

Unfortunately, when a further effect of finite temperature , namely, the damping in quark propagation, was incorporated by Gavela et al [8] in the original calculation of Ref. [7], the baryon asymmetry again turned out to be too small in the minimal model. One is thus led to consider its nonminimal extensions.

In the nonminimal models, the propagation of the heavy t quark is usually supposed to yield the largest contribution to the baryon assymmetry. Its mass being large compared to the temperature ($\sim$ 100 GeV), it does not undergo any significant finite temperature modification, at least in the broken phase. On the other hand, if we consider a lighter quark like the b quark, its dispersion relation is strongly modified by such temperature effect, as it acquires a large chirally invariant effective mass and a decay rate. The resulting formula for the baryon asymmetry generated by the b quark is quite different in structure from that by the t quark. As a result, one finds that provided the CP violation within the wall is large enough (but within the allowed limit), the observed baryon asymmetry can be reproduced by the b quark alone.

In Sec 2 we present a simplified version of the baryon asymmetry calculation by Nelson et al [5], who considered the propagation of t quark because of its large mass. In Sec 3 we then sketch an analogous calculation for the lighter b quark, taking finite temperature effects into account. Sec 4 contains our comments regarding the magnitudes of CP violation by the t quark and by the b quark within the wall needed to reproduce the observed baryon asymmetry. A criticism raised against the present way of incorporating the decay rate in the problem is also mentioned.

2. Heavy quark propagation

Consider the propagation of the (heavy) t quark in the electroweak plasma. Its propagation will not be significantly modified by the plasma. In particular, the damping effect may be neglected. The appropriate Lagrangian is then

$$\mathcal{L} = iR^\dagger(\partial_0 + \sigma.\nabla)R + iL^\dagger(\partial - \sigma.\nabla)L + m(x)L^\dagger R + m^*(x)R^\dagger L, \quad (1)$$

where R and L are the right and the left handed components of the Dirac wave function in Weyl basis,

$$R = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \qquad L = \begin{pmatrix}\psi_3\\ \psi_4\end{pmatrix}$$

The first two terms in (1) are just the decomposition of the Dirac Lagrangian in Weyl basis, to which are added the last two terms representing the complex, space-dependent Higgs mass.

Consider the one dimensional problem of propagation along the z-axis, normal to the bubble wall. Then the equation of motion derived from the Lagrangian (1) split into two independent sets. Define

$$\Phi = \begin{pmatrix} \psi_1 \\ \psi_3 \end{pmatrix} \qquad \Phi' = \begin{pmatrix} \psi_4 \\ \psi_2 \end{pmatrix}$$

and consider solution of positive energy E. Then the equations of motion reduce to

$$\frac{d}{dz}\Phi = iQ(z)\Phi, \tag{2}$$

where

$$Q(z) = \begin{pmatrix} E & m^\star(z) \\ -m(z) & -E \end{pmatrix}. \tag{3}$$

and a similar one for Φ' with m replaced by $m^\star$. The current along the z-axis carried by the components ψ_1 and ψ_3 of Φ is

$$j_z = \Phi^\dagger \sigma_3 \Phi, \tag{4}$$

σ_3 being the third Pauli matrix.

The planar bubble wall has a finite thickness, extending from $z = 0$ to $z = z_0$, separating the broken phase ($z > z_0$) from the unbroken phase ($z < 0$). The real part of the Higgs induced mass $m(z)$ rises from zero in the unbroken phase through the bubble wall to the (almost) zero temperature mass m_0 in the broken phase. The imaginary part is non-zero only within the bubble wall. Their actual shapes will be conveniently chosen later.

In the unbroken phase ($m = 0$), the components ψ_1 and ψ_3 (also ψ_2 and ψ_4) decouple. ψ_1 and ψ_3 belonging to chirality +1 and −1 describe plane wave propagation to the right and to the left along the z-axis respectively. For ψ_2 and ψ_4 the direction of motion is reversed.

In the broken phase ($m(z) = m_0$), the wave function of the quark moving along the positive z-direction is

$$\begin{pmatrix} c \\ -s \end{pmatrix} e^{ipz} \tag{5}$$

where

$$c = \sqrt{\frac{E+p}{2p}}, \qquad s = \sqrt{\frac{E+p}{2p}}$$

Within the wall, we solve (2) perturbatively. Put

$$\Phi(z) = e^{iEz\sigma_3}\Psi(z), \qquad 0 \leq z \leq z_0,$$

$\Psi(z)$ will then satisfy

$$\frac{d\Psi}{dz} = iR(z)\Psi.$$

$R(z)$ has only off-diagonal elements,

$$R(z) = \begin{pmatrix} 0 & M^*(z) \\ -M(z) & 0 \end{pmatrix}$$

where $M(z) = m(z)e^{2iEz}$. We now convert it into an integral equation,

$$\Psi(z) = \Phi(0) + i\int_0^z R(z')\Psi(z')dz'.$$

It has an iterative solution, $\Psi(z) = \Sigma(z)\Phi(0)$, where

$$\Sigma(z) = 1 + i\int_0^z dz' R(z') - \int_0^z dz' \int_0^{z'} dz'' R(z')R(z'') + \cdots$$

We shall actually need the solution for $\Phi(z)$ at $z = z_0$,

$$\Phi(z_0) = e^{iEz_0\sigma_3}\Sigma(z_0)\Phi(0) \equiv \Omega(z_0)\Phi(0). \tag{6}$$

Writing

$$\Omega(z_0) = \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix}$$

we get

$$\alpha = F(1 + \int_0^{z_0} dz' \int_0^{z'} dz'' M^*(z')M(z'') + \cdots) \tag{7}$$

$$\beta = iF(\int_0^{z_0} dz' M^*(z') + \cdots) \tag{8}$$

with $F = e^{iEz_0}$. The matrix $Q(z)$ being traceless, $\Omega(z_0)$ is unimodular: $\alpha\alpha^* - \beta\beta^* = 1$.

As already mentioned, simple extensions of the Higgs sector of the standard model can provide an additional source of large CP violation for baryogenesis. In the standard model with a single Higgs doublet, the expectation value of the Higgs field is real everywhere during the phase transition. But in multi-Higgs models, some of the components acquire complex space dependent values within the bubble wall, leading, in turn, to complex space dependent mass functions for the quarks having Yukawa couplings to those multiplets. These functions can, in principle, be calculated from the model considered but will depend on the many unknown Higgs self-couplings. Here we avoid this problem by assuming a simple but anticipated form for the mass function for the quark,

$$m(z) = \frac{m_0}{z_0} z + i \frac{\delta}{z_0^2} z(z_0 - z), \tag{9}$$

within the bubble wall. The parameter δ relates to the CP violation in the model. Then α and β are obtained in terms of the dimensionless variables $m_0 z_0, \delta z_0$ and $E z_0$. In the following we assume that the wall thickness z_0 to be small enough such that they are less than unity. Then to first order in these variables we have,

$$\alpha = 1 + iEz_0, \qquad \beta = \frac{i}{2} m_0 z_0 + \frac{1}{6} \delta z_0 \tag{10}$$

It is now easy to calculate the reflection and transmission coefficients for particles incident on the wall. Consider a right-handed fermionic quasiparticle incident on the bubble wall from the unbroken phase. Noting the reversal of chirality after reflection at the wall, the incident wave (of unit current at $z = 0$) and the reflected wave of amplitude r, say, is given by

$$\Phi(z) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ikz} + \begin{pmatrix} 0 \\ r \end{pmatrix} e^{-ikz}, \; z \leq 0 \tag{11}$$

On the right (broken phase), we have only the transmitted wave of amplitude t, say. From (5) we get

$$\Phi(z) = t \begin{pmatrix} c \\ -s \end{pmatrix} e^{ip(z-z_0)}, \; z \geq z_0. \tag{12}$$

To find the unknown amplitudes we insert (11) and (12) in the matching condition (6) to get,

$$\begin{pmatrix} c \\ -s \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix} \begin{pmatrix} 1 \\ r \end{pmatrix},$$

giving

$$r = -\frac{s\alpha + c\beta^*}{c\alpha^* + s\beta}, \qquad t = \frac{1}{c\alpha^* + s\beta}, \qquad |r|^2 + |t|^2 = 1$$

For the antiparticles the amplitudes, denoted by $\bar{r}$ and $\bar{t}$, are obtained by changing the sign of δ.

We can now calculate the net baryonic current due to reflection from and transmission through the barrier. The reflected current due to incidence from the unbroken phase is

$$\int \frac{dk}{2\pi} n^u(k)(|r|^2 - |\bar{r}|^2),$$

where n^u is the fermionic number density in the unbroken phase. One can similarly calculate the transmitted current in the unbroken phase due to incidence from the right ,

$$\int \frac{dp}{2\pi}\frac{\partial E}{\partial p} n^b(-p)(|t|^2 - |\bar{t}|^2),$$

n^b being the fermion number density in the broken phase. Adding the above two currents, we finally get the baryonic current in the unbroken phase as

$$J = \int \frac{dk}{2\pi}(n^b(-p) - n^u(k))(|t|^2 - |\bar{t}|^2). \tag{13}$$

We now evaluate J. Using (10) we get to leading order

$$|t|^2 - |\bar{t}|^2 = -\frac{4}{3} m_0 \delta z_0 \frac{p}{(p+k)^2} \tag{14}$$

To evaluate the density difference, we start from the invariant form of the density function

$$n = \frac{1}{e^{\beta p.v} + 1}$$

Here β is the inverse temperature of the medium in the frame where it is at rest. p_μ and v^μ are the energy-momentum 4-vector of the particle and the 4-velocity of the medium. We work in the wall rest frame in which $v^\mu = \gamma(1, v)$, $\quad \gamma = \frac{1}{\sqrt{1-v^2}}$. In the following we assume small wall velocity and retain only terms linear in v.

In the unbroken phase $p_\mu \equiv k_\mu = (|k|, k)$, when

$$n^u(k) \quad \frac{1}{e^{\beta k(1-v)} + 1}$$

In the broken phase $p_\mu = (E, p)$ with $E^2 = k^2 = p^2 + m_0^2$. The density function becomes

$$n^b(-p) = \frac{1}{e^{\beta(k+pv)} + 1}$$

Then we get

$$n^b(-p) - n^u(k) = -A\beta v(k+p)$$

where

$$A = \frac{e^{\beta k}}{(e^{\beta k}+1)^2} \quad 0.1$$

It is now easy to make an order of magnitude estimate of the currents. Formally the k -integral in (13) extends from 0 and ∞. But the integrands are highly damped at higher values of k, not just because of the presence of the density functions; the transmission coefficients for a realistic (i.e. smooth and finite width) bubble wall would have fallen exponentially for even lower k values. We set E_0 as a reasonable upper limit for the integral. The low momentum approximation on which the effective Lagrangian (1) is based, should admit this upper limit. Then collecting results, we get

$$\begin{aligned} J &= \frac{2}{15}\beta v m_0 \delta z_0 \int_{m_0}^{3m_0/2} \frac{dk}{2\pi} \frac{p}{p+k} \\ & \quad \frac{1}{90\pi}\beta v m_0^2 \delta z_0. \end{aligned}$$

Finally the baryonic density n_B in the broken phase is obtained from the steady state solution to the rate equations in the two phases [7]. For small bubble wall velocity the result is

$$n_B = Jf \tag{15}$$

where f is a given function of the diffusion coefficients for quarks and leptons, the wall velocity and the sphaleron induced baryon number violation rate. The estimate for f is $10^{-3} \leq f \leq 1$ [7]. Noting the one dimensional entropy density $s = 73\pi/3\beta$, the baryon to entropy ratio is thus obtained as

$$(n_B/s)_t = \frac{f}{2190\pi^2}(\beta m_0)^2 v\delta z_0 \simeq 1.4 \times 10^{-5}(f\delta z_0)_t \tag{16}$$

where we have assumed $m_0 = 170GeV, \beta = 10^{-2}GeV^{-1}$ and $v = 0.1$. Here we have isolated the model dependent parameters namely f, δ and z_0. The parameter δ should satisfy $\delta < 170GeV$. Although z_0 can be as large as $10/TGeV^{-1}$ [9], the validity of our treatment requires that z_0 be small enough such that $6\gamma z_0 < 1$. Otherwise the calculation will violate unitarity. Clearly there is wide room for reproducing the observed value of $n_B/s \sim 10^{-10}$.

3 Light quark propagation

The propagation properties of a light quark of mass m at temperature T $(m < T)$ will be greatly modified due to its interaction with the plasma. It will acquire a large chirally invariant self energy, the real part of which is the effective mass E_0 and the imaginary part is the decay rate γ. Considering only strong interaction, the leading contribution to E_0 and γ are the same for both the right (R)- and the left (L)- handed particles,

$$E_0 = (2\pi\alpha_s/3)^{1/2}T \simeq .5T$$

and

$$\gamma = .15\alpha_s T \simeq .2T$$

with $\alpha_s = 0.12$ at the Z boson mass. For excitations close to E_0, the effective Lagrangian incorporating the altered dispersion relation is now [7,8]

$$\mathcal{L} = 2iR^\dagger(\partial_0+\frac{1}{3}\sigma\cdot\nabla+iE_0+\gamma)R+2iL^\dagger(\partial_0-\frac{1}{3}\sigma\cdot\nabla+iE_0+\gamma)L+mL^\dagger R+m^*R^\dagger L, \tag{17}$$

The one dimensional motion of these quasiparticles can then be studied in the same way as in the previous section, with the replacement of Q in eqn(2) by

$$Q(z) = 3\begin{pmatrix} E - E_0 + i\gamma & m^*(z)/2 \\ -m(z)/2 & -(E - E_0 + i\gamma) \end{pmatrix} \tag{18}$$

The dispersion relations satisfied by these quasiparticles are very different from the zero temperature dispersion relation considered earlier. In the unbroken phase ($m = 0$), the components ψ_1 and ψ_3 (also ψ_2 and ψ_4) decouple. Consider (damped) plane wave along z direction, $\psi_{1,3} \sim e^{iKz}, K = k + i\Gamma_u, k > 0$. They satisfy the the two-branch ($\pm$) dispersion relations ,

$$E_\pm = E_0 \pm \frac{k}{3}, \qquad \Gamma_u = \pm 3\gamma, \tag{19}$$

In the broken phase, again considering damped plane wave along z-direction, $\Phi \sim \chi e^{iPz}, P = p + i\Gamma_b, p > 0$. one gets the dispersion relation,

$$E_\pm = E_0 \pm \frac{p}{3} g(p), \qquad \Gamma_b = \pm \gamma g(p), \tag{20}$$

where $g(p) = \sqrt{1 + \frac{m_0^2/4}{\gamma^2 + p^2/9}}$. Note that the presence of damping ($\gamma \neq 0$) removes any gap between the normal and the abnormal branches, which exists for $\gamma = 0$. The spinor χ is obtained as ,

$$\chi = \begin{pmatrix} cosh\theta . e^{i\phi} \\ -sinh\theta . e^{-i\phi} \end{pmatrix}, \tag{21}$$

with $cosh\theta = [(E - E_0 + p/3)/(2p/3)]^{1/2}$ and $4i\phi = ln[(p/3 + i\gamma)/(p/3 - i\gamma)]$. The presence of non-zero damping brings in also the phase ϕ.

The previous calculational procedure may now be generalised incorporating the damping rate and the propagation of the quark through both the brances of the dispersion relation. This calculation is described in Ref.[10]. We find that the baryon to entropy ratio is,

$$(n_B/s)_b = \frac{f}{5810\pi^2} \beta^2 E_0 m_0 v \delta z_0 \simeq 4 \times 10^{-8} (f \delta z_0)_b, \tag{22}$$

with $m_0 = 5 GeV, \beta = 10^2 GeV^{-1} and v = 0.1$. Thus if $(\delta)_b$is about two orders of magnitude higher than $(\delta)_t$, the baryon asymmetry created by the b quark and by the t quark are comparable.

4 Conclusion

Here we have made a comparative study of the propagation of the heavy t quark and of the light b quark across the bubble wall during the electroweak phase transition. Although the general procedure is the same for both the quarks, they differ with respect to finite temperature corrections. For the t quark these corrections are not large and so may be ignored. But for the b quark, the finite temperature correction overwhelms the zero temperature dispersion relation and must be taken into account. Comparing the expressions for the baryon asymmetry generated by the t quark and by the b quark, one sees that the incorporation of the effective mass compensates for the smallness of the b quark mass to some extent. Also the damping does not significantly affect the final expression for the baryon asymmetry due to the b quark [11]. In the absence of a better knowledge of CP violation by the quarks (as measured by its imaginary part within the wall relative to the physical mass), we conclude that, while both the quarks can reproduce the observed asymmetry, the t quark requires smaller CP violation than is needed by the b quark. Pushing to the limit, we may say that even if the CP violation for the t quark turns out to be zero, the b quark alone could reproduce the observed baryon asymmetry.

Finally we remark that this way of incorporating the decay rate in this formalism may not be a physically correct procedure. While Gavela et al [8] take the effective mass and the decay rate on the same footing (as we do here), Shaposhnikov [12] thinks that while the effective mass alters the dispersion relation, the decay rate serves the same purpose as that of the collision integral in the Boltzmann equation. In this work we show that although the baryon asymmetry turns out to be too small in the minimal standard model when the two quantities are treated on the same footing, the same mechanism can give rise to the observed asymetry in the nonminimal models.

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